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SURFACE PHENOMENA CAUSING BREAKDOWN OF FALLING LIQUID FILMS DURING HEAT TRANSFER

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Abstract—This paper covers the case of a sub-cooled falling liquid film and its breakdown during heat transfer, and is a preliminary study to that of burnout in the case of two phase flow. The hydrodynamic changes of the film due to surface tension difference forces are considered with the aid of a rigorous mathematical analysis for isothermal systems. A correlation, involving the mathematical statement for the wave-number (λ/a_0) for isothermal flow, is derived to predict flow rates of water when breakdown occurs for given heating conditions, and is

$$0{\cdot}0058\left(\frac{\lambda}{a_0}\right)\left(\frac{\Delta\sigma}{\sigma_f}\right)\left(\frac{h_b}{1000}\right)^{6\cdot89} = \left(\frac{4\rho Q}{\mu}\right)_f^{-0\cdot68}$$

NOMENCLATURE

a,	film thickness of flow in wave motion;
a_0,	mean film thickness of flow in wave motion;
f,	function;
g_x,	acceleration due to gravity;
h_b,	heat-transfer coefficient for heating fluid based on bulk temperature t_b;
h_f,	heat-transfer coefficient of film;
k,	phase velocity of waves;
m,	film thickness for fully developed laminar flow;
n,	$\left(=\frac{2\pi}{\lambda}\right)$ frequency;
p,	capillary pressure;
Q,	liquid flow rate per unit length of wetted perimeter;
Re,	Reynolds number;
t,	time;
u_0,	mean velocity of film;
u,	point velocity of film in x-direction;
v,	point velocity of film in y-direction;
x,	vertical co-ordinate in direction of flow;
X_{cr},	length of laminar flow before inception of waves;
y,	ordinate perpendicular to direction of flow;
Z,	$\left(=\frac{k}{u_0}\right)$ ratio of phase velocity to mean stream velocity;
K, A, B, C, D,	constants.

Greek symbols

α,	amplitude of waves;
δ,	$\left(=\frac{\sigma}{\rho}\right)$ kinematic surface tension of liquid;
μ,	dynamic viscosity of liquid;
ρ,	mass density of liquid;
σ,	surface tension of liquid;
$\Delta\sigma$,	surface tension difference based on difference between the surface tension of water at film inlet temperature and the surface tension of water at the bulk heating temperature;
λ,	fundamental wavelength of wave motion;
ν,	kinematic viscosity of liquid;
∇,	$\left(=\Phi-\frac{\lambda}{a_0}\right)$ distortion parameter;
Φ,	$\left(=\frac{\lambda}{a_0}\exp\left[f\left(\frac{\lambda}{a_0}\right)\right]\right)$ flow pattern parameter.

Subscript

f,	film at inlet temperature.

INTRODUCTION

THIS paper covers the introductory work carried out in the investigation of the breakdown phenomenon of liquid films due to heat transfer when flowing vertically downwards on the outside of a copper tube. This topic is a first step in the appreciation of "burnout" in boiler tubes where two phase flow exists; further, as there is similarity between heat and mass transfer, this paper may be useful to chemical engineers where low flow rates occur in distillation.

ISOTHERMAL FLOW

Essential to this work is that of isothermal systems which have received rigorous mathematical analysis. In Fig. 1 is shown an idealized

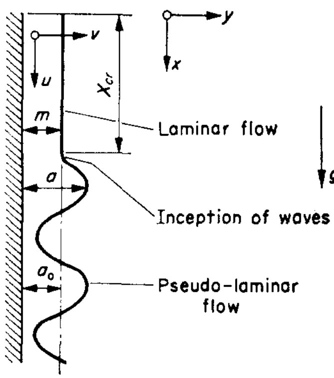

FIG. 1. Isothermal flow model.

model of the flow. Nusselt [1] obtained, assuming pure laminar flow, expressions for the mean velocity of flow and film thickness m:

$$m^3 = \frac{3\mu Q}{\rho g_x}. \qquad (1)$$

In this classical approach, surface tension forces are not introduced into the hydrodynamic theory. However, Kapitza [2] has shown that surface tension forces are important where thin films are concerned even when small curvature of the free surface occurs. In simple terms, the gain in the free surface energy due to increase in surface area caused by disturbance enables work to be done on the film and wave motion ensues. The Reynolds number, $(4\rho Q/\mu)$, is above 4 to 25 when these waves occur; below this range pure laminar flow exists.

The fundamental momentum equation in the x-direction is given by:

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + g_x + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

In this equation the velocity component v is neglected because the wavelength to film thickness ratio is large.

To establish the equation for the case of wave motion, mean values of u and u^2 for a cross-section of the film are substituted for the point values of u and u^2. These mean values and the derivatives with respect to time t, x and y are determined from a velocity profile which is parabolic and the following statements:

$$a = a_0(1 + \phi), \qquad (3)$$

and

$$\phi = f(x - kt). \qquad (4)$$

The pressure gradient is considered to be due to capillary forces alone so that:

$$\frac{\partial p}{\partial x} = -\sigma\frac{\partial}{\partial x}\left\{\frac{\partial^2 a/\partial x^2}{[1 + (\partial a/\partial x)^2]^{\frac{3}{2}}}\right\}. \qquad (5)$$

Hence a first approximate linearized equation is obtained and shown to be:

$$\delta a_0 \frac{\partial^3 \phi}{\partial x^3} + u_0^2(Z - 1)(Z - 1{\cdot}2)\frac{\partial \phi}{\partial x} + 3\left(g_x - \frac{Z\nu u_0}{a_o^2}\right)\phi + \left(g_x - \frac{3\nu u_0}{a_o^2}\right) = 0 \qquad (6)$$

which gives the fundamental frequency n to be

$$n^2 = \frac{u_o^2}{a_0\delta}(Z - 1)(Z - 1{\cdot}2). \qquad (7)$$

An F-function of the form

$$F = \frac{1}{\lambda}\int_0^\lambda \frac{(1 + Z\phi)^2}{(1 + \phi^3)}\,dx = \frac{\overline{(1 + Z\phi)^2}}{(1 + \phi^3)} \qquad (8)$$

establishes

$$a_0^3 = \frac{3\mu Q}{\rho g_x} F \qquad (9)$$

when the energy dissipated due to viscous forces balances the work done due to gravity forces, hence:

$$n^2 = (Z - 1)(Z - 1{\cdot}2)\frac{1}{3F}\frac{g_x Q}{\delta\nu}. \qquad (10)$$

Now the value of F will be a minimum for the maximum amplitude of oscillations occurring at steady conditions. Portalski [3] has obtained values of this F-function to give

$$F\min = 0{\cdot}83, \qquad (11)$$

and

$$Z = 2{\cdot}48. \qquad (12)$$

Substitution of these values in equations (9) and (10) leads to the statement of the wave number (λ/a_0) of the fundamental

$$\frac{\lambda}{a_0} = 5{\cdot}33\left(\frac{\mu\sigma^3}{\rho^4 g_x Q^5}\right)^{\frac{1}{6}} \qquad (13)$$

Brooke-Benjamin [4] has derived in his theoretical analysis of film flow the same non-dimensional group ($\mu\sigma^3/\rho^4 g_x Q^5$) which can be said to describe the wave motion. A linearized theory shows that there is a class of wavelike disturbances which undergo unbounded amplification. The inception of the wave motion following an apparent pure laminar flow regime is the visible appearance of instability. The presence or absence of surface tension does not alter this general conclusion. The limitation of the theory is that it applies only for $Re < 20$ whilst that of Kapitza [2] covers the whole range of Reynolds numbers. If the wave number of the most unstable wave is defined in the same way as Kaptiza's,

$$\frac{\lambda}{a_0} = 5{\cdot}618\left(\frac{\mu\sigma^3}{\rho^4 g_x Q^5}\right)^{\frac{1}{6}}. \qquad (14)$$

Binnie [5], in an experiment concerned with the onset of wave formation, has given an average value of the wavelength of the most unstable wave as 0·45 inch for a Reynolds number of 17·6; Brooke-Benjamin compared a theoretical value of 0·42 inch. Experimental work by Portalski [3], in conjunction with his mathematical analysis of isothermal systems, gave favourable results. To quote, an experimental value of the phase velocity Z of $\sim 2{\cdot}5$ is compared with a theoretical value of 2·48.

Recently Hartley and Murgatroyd [6] have studied theoretically thin liquid films flowing isothermally down a vertical surface where a dry or unwetted region exists. Criteria for the complete re-wetting of the surface are suggested.

NON-ISOTHERMAL FLOW

Norman and McIntyre [7] carried out an investigation of the breakdown phenomenon solely due to heat transfer, where the film flowed on the inside of a copper tube of length 3¼ inch. The distortion of the flow pattern, it is suggested, is due to surface tension differences occurring in the surface of the film. These differences arise from uneven heating as a result of variations in thickness of the film caused by rippling. The region of greater thickness will have the less temperature so the subsequent higher surface tension force will draw liquid from the thinner region. Finally liquid deficiency in the thinner region occurs and a dry patch on the tube surface appears.

Hsu, Simon and Lad [8] in their investigation into liquid film breakdown during heat transfer considered it essentially as an entrance region problem. The liquid film flowing down the surface of a vertical tube passes over a heating surface 13 in long and 15 in from an upper reservoir. As the inception of waves for the Reynolds number range considered is always prior to the heating surface the investigation is solely concerned with wavy motion. From the leading edge of the heating section there is an entrance length required for the thermal boundary layer to develop to the same thickness as the liquid film. Before the thermal boundary layer is fully developed the temperature of the free surfaces can be taken as the inlet temperature. When irregularities occur in the film thickness and the thermal boundary layer is at least fully developed for the thicker region, temperature differences exist as the free surface temperature of adjacent thinner regions will be greater. If these temperature differences are capable of sustaining "steady" and not fluctuating surface tension difference forces the thermocapillary effect is to distort the film and make it thinner. If the heat flux is high enough a dry patch forms on the tube surface. The entrance length for a given film thickness is derived analytically by solving the heat flow equation in which the heat input is equated to the rise in enthalpy of the film determined by the integral boundary layer method. The velocity profile in the film is assumed to be that of von Kármán–Nikuradse. Further, a *simplified theory particular to the* boundary conditions stated above shows that the energy input to the actual distorted length of film ending in a dry patch is constant for a liquid film of given Reynolds number, subjected to different heat fluxes. Experiment substantiates this theory very favourably, and also shows that with increasing Reynolds number the energy input to the film to cause a dry patch needs to be increased.

Norman and Binns [9] and Ponter and Thornley [10] have illustrated the effect of mass transfer on liquid films. Rippling causes different concentrations on absorption of a vapour such as ammonia to produce surface tension differences which cause breakdown.

APPARATUS AND EXPERIMENTAL WORK

The apparatus used in the experimental work is shown in Fig. 2. Flow down the outside of the vertical copper tube of 1 inch inside diameter was achieved by means of an upper reservoir in the base of which was an annulus through which water flowed (Fig. 3). The film drained into a sink some 6 ft below and was returned to a tank (Fig. 4) where the head was kept constant by an external feed and overflow. This feature enabled the excess heat picked up by the film in flowing down the outside of the tube to be extracted

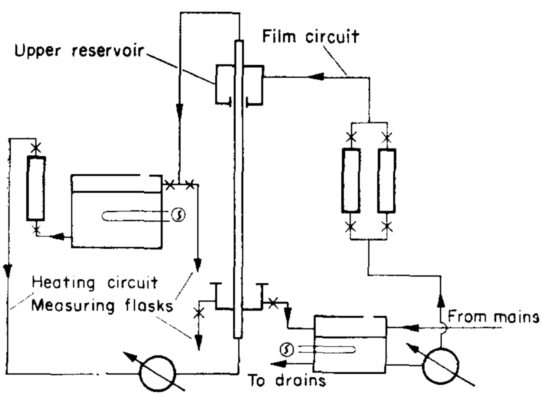

FIG. 4. Line diagram of apparatus.

from the loop. Then water could be pumped from the tank to the upper reservoir at a sensibly constant temperature. Heating was by hot water flowing counterwise at a high rate of flow of the order of 1 lb/s for which the temperature drop across the full length of 9 ft of tubing was 1 degF for low flows to 5 degF for maximum flows of the film. As the working length of the tube was at the most 9 in, i.e. the furthermost distance from the annulus where breakdown occurred, the bulk heating temperature could be assumed constant for this length. A rotameter was inserted in the heating circuit to ensure the heat-transfer coefficient was kept constant during test runs. Thermometers and thermocouples were located across the full tube length, and also in the upper reservoir to determine film inlet conditions. Whilst the calibrated thermometers were used to obtain temperature readings the thermocouples were mainly used because of their sensitivity to detect immediately any temperature movement from set conditions. This facility proved valuable when unsteady conditions were set up in reducing the film flow rate to obtain film breakdown.

FILM FLOW CHARACTERISTICS ON THE OUTSIDE OF THE COPPER TUBE

In Fig. 3 is shown isothermal flow viewed from below the annulus, the source of the film. The width of the annulus is greater than the thickness of a falling film given by the Nusselt theory [1], reference equation (1), for the maximum Reynolds number $(4\rho Q/\mu)$ of the experimental work. Consequently there exists a vena-contracta at the annulus and the thickness of the film will approximate to the Nusselt theory though not completely as the velocity profile is not here fully developed.

The illustrations Figs. 3 and 5 show the effect on the flow pattern of Reynolds number. At low values of Reynolds number the wave number (λ/a_0), reference equation (13), is associated with capillary waves, and at high values of Reynolds number the wave number (λ/a_0) describes gravity waves of greater frequency. Before the inception of wave motion there exists a laminar flow region, Fig. 1, the length X_{cr} of which depends upon Reynolds number, and its variation is shown in Fig. 6.

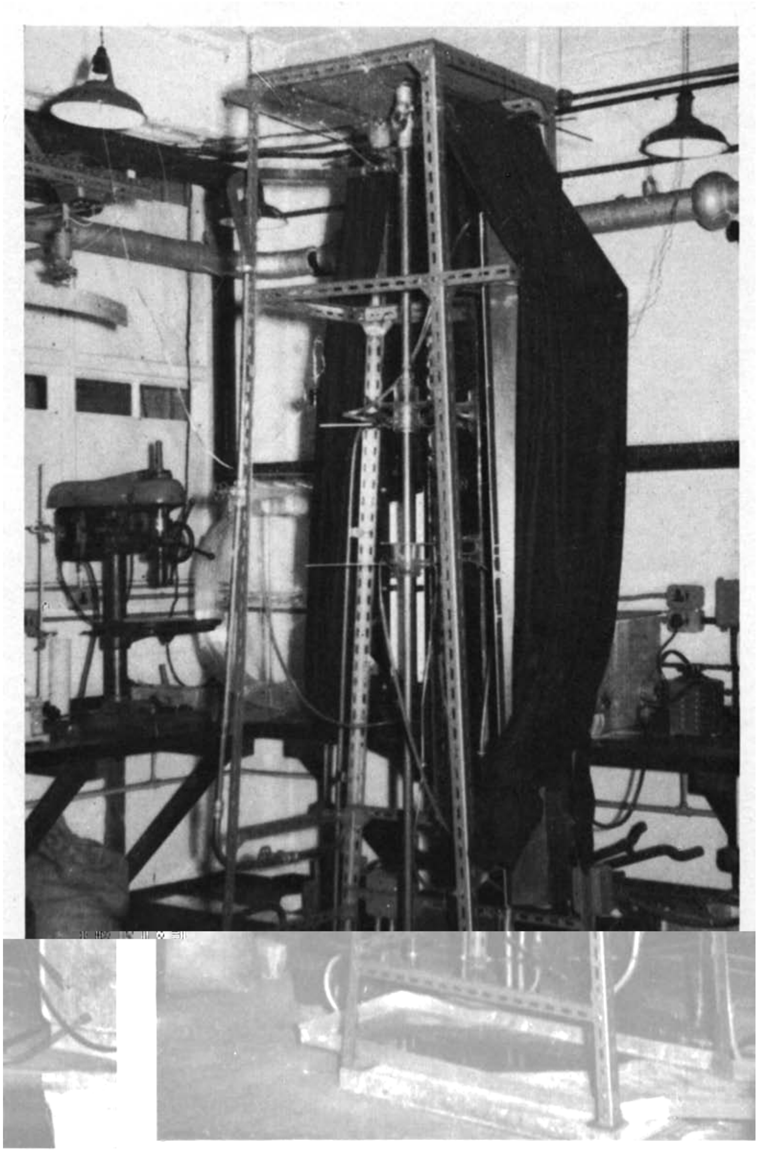

FIG. 2. Apparatus.

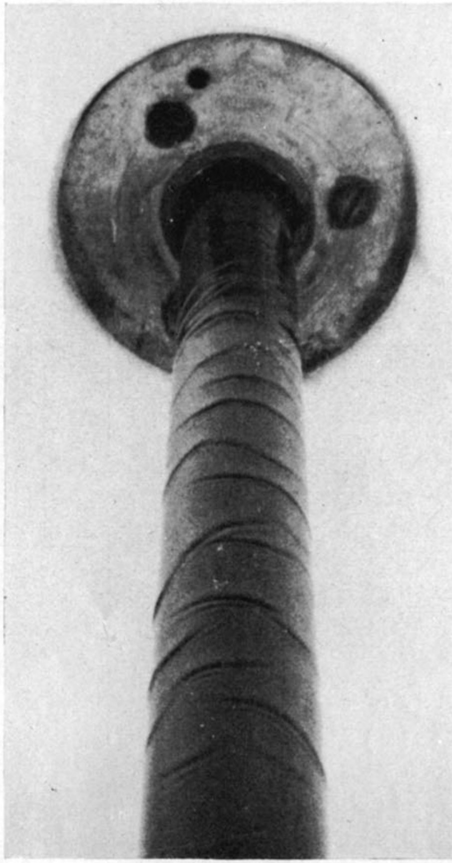

FIG. 3. Isothermal flow, $Re = 100$.

FIG. 5. Isothermal flow, $Re = 500$.

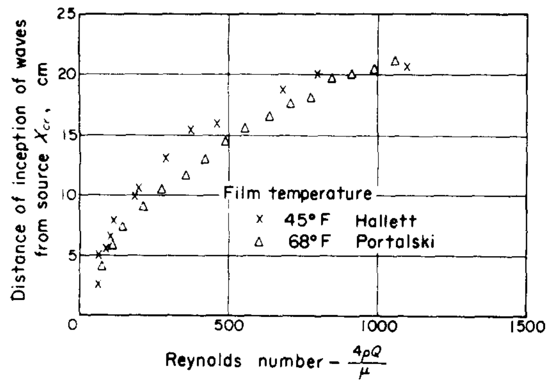

FIG. 6. Extent of pure viscous flow before wave inception.

At the inception of wave motion the wave number (λ/a_0), given by Portalski, is predominant, but waves of multiple periodicity develop as shown in Fig. 3 to cause coalescing of waves, and the resulting flow pattern is irregular and complicated. Whilst the mathematical analysis applies to the case of vertical plane surfaces, the analysis as outlined in the introduction can be applied to the case in hand as the ratio of the film thickness to pipe diameter is small.

Lack of geometric symmetry induced an irregular flow pattern which had to be mitigated as much as possible. The copper tube was vertically placed by means of a plumb line; however, the major issue was ensuring concentricity of the annulus with the copper tube. The effect of eccentricity of the annulus is to vary the value of the local Reynolds number ($4\rho Q/\mu$) around the circumference of the tube. The subsequent variation in the length of pure laminar flow causes the line of inception of the waves to depart from the horizontal plane which is required to provide the initiation of regular wave pattern.

The significance of eccentricity of the annulus position can be appreciated from the following analysis. From Fig. 6 the variation in the position of the inception of waves with Reynolds number is given to be

$$\frac{\Delta X_{cr}}{\Delta Re} \simeq \frac{15}{500} \text{ cm}$$

for $Re < 500$ where X_{cr} = length of laminar region.

Now $$m^3 = \frac{3\mu Q}{\rho g_x}$$

as $$Re = \frac{4\rho Q}{\mu}$$

and $$\frac{4\rho Q}{\mu} = \frac{4 g_x m^3 \rho^2}{3\mu^2}.$$

Then $$\frac{d(Re)}{dm} = \frac{4 g_x m^2}{\nu^2}$$

$$\therefore \frac{\Delta X_{cr}}{\Delta m} = \frac{4 g m^2}{\nu^2} \times \frac{15}{500}$$

for water at 68°F.

$$\frac{\Delta X_{cr}}{\Delta m} = \frac{4 \times 981 \times (m)^2}{(1{\cdot}05 \times 10^{-2})^2} \times \frac{15}{500}$$

For $Re = 500 \quad m = 0{\cdot}035$ cm.

Then

$$\frac{\Delta X_{cr}}{\Delta m} = 1300. \tag{15}$$

This equation reveals the remarkable sensitivity of the movement of the line of inception of waves with variation in thickness of film or eccentricity of the annulus. In practice it was found impossible to achieve a horizontal plane for the initiation of waves, as some eccentricity between the pipe wall and the annulus could not be eliminated.

MODES OF ACHIEVING HEAT TRANSFER

The falling film can be heated by various methods, but hot water flowing counterwise at a high rate is, in the author's opinion, the best in the circumstances. Two other methods were considered and rejected after reference to an ideal mathematical model which has a constant wall temperature and known heat flux input to the film. Heating by electrical means enables the flux input to be determined. However, because the film is seriously distorted during heat transfer, the wall temperature will vary considerably from thin to thicker film regions. Consequently the wall temperature cannot be used as a reference, which is necessary for the correlation of experimental data. Condensing steam has been used for heating purposes to produce known and uniform wall temperature conditions. It is assumed that the saturation temperature is uniform through the film. This is a reasonable assumption at low flow rates of the condensate, but at high flow rates, when the flow rate of the film outside the copper tube is large, an unknown and significant temperature profile due to sub-cooling can exist. Apart from the fact that the wall temperature is then unknown, accurate determination of the heat flux and/or the condensing steam film coefficient is not possible in this application.

The merit of the mode of heating used in this project is that the bulk heating temperature is sensibly constant over the working range of the apparatus due to the high rate of flow of the heating medium. Also the related heat-transfer coefficient for the turbulent pipe flow can be determined. Whilst this does not entirely meet the requirements of the mathematical model at least these references are sound to permit correlation of experimental data.

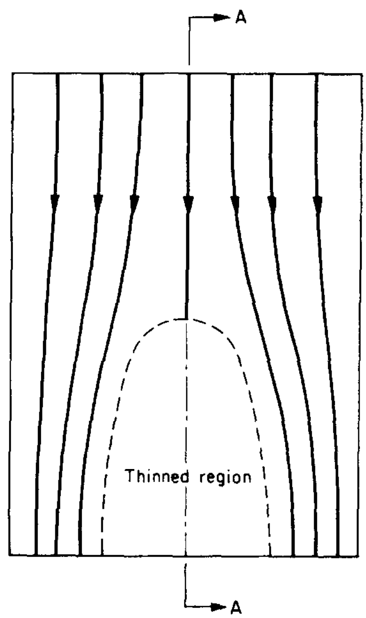

FIG. 7. Development of distorted film in breakdown region.

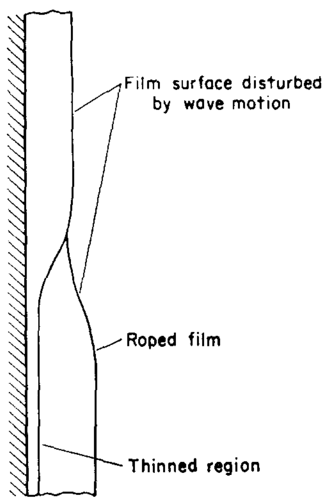

FIG. 8. Cross-sectional view A–A.

TEST PROCEDURE

For given heating conditions fixed by the bulk heating temperature and the mass flow rate of the hot water, the falling film at a fixed inlet temperature was reduced in flow rate slowly and in stages. Distortion of the film caused roping, and the film thinned as shown in Figs. 7 and 8 until eventually a dry patch occurred on the tube surface. The flow rate at breakdown was measured by tapping off the flow from a lower reservoir on the tube into a graduated flask. A constant reading on a low flow rate rotameter ensured steady conditions during the measurement of the minimum wetting rate. This test was repeated for a range of film inlet temperatures. Sets of readings were subsequently obtained for different heating conditions.

In order to gain repeatability of results, particular attention had to be given to the surface condition of the tube. Buffing produced a smooth matt finish and chemical cleaning inhibited the effect of impurities. The magnitude of the flow rates at breakdown are sensibly affected if the surface is initially wetted or not before experimental work. If the surface is not completely wetted then effectually two air–water interfaces exist. The copper–air–water regime would provide a variation in surface tension effects to cause appreciable scatter in results. Water was allowed to run overnight over the copper surface so that water molecules could be uniformly adsorbed in the surface to displace the air.

EXPERIMENTAL RESULTS

Isothermal flow

In Fig. 6 is shown the variation in length X_{cr} of laminar flow before inception of waves with Reynolds number. Generally there is good agreement with Portalski [3] except for the middle range of Reynolds number where the rate of change X_{cr} with $4(\rho Q/\mu)$ is significant. Eccentricity of the annulus producing variations in the local values of Q may account for this difference. Hence X_{cr} should strictly be plotted against the local value of Re instead of that based on the mean value of the specific flow rate Q.

Non-isothermal

In Fig. 9 is shown minimum wetting rates (cm³/s cm) plotted against film inlet temperature for a bulk heating temperature of 200°F. Though agreement exists with Norman and

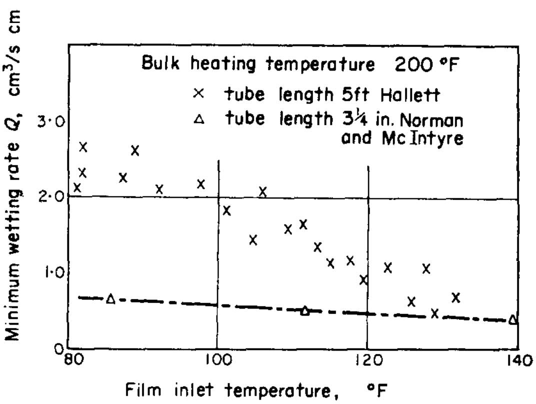

FIG. 9. Variation in minimum wetting rates with film inlet temperature for constant bulk heating temperature.

McIntyre [7] at high film inlet temperatures, there is a marked difference at low film temperatures. It is noted in the case of the latter's work that film flowed on the inside of a 1 inch diameter copper tube of length 3¼ inch and the heating was achieved again by hot water flowing at a high rate to ensure constant bulk heating temperature conditions. In the present work, minimum wetting rates were also obtained for bulk heating temperatures of 140°F, 160°F and 180°F, varying in each case the film inlet temperature from 55°F to within 40–50 degF of the bulk temperature.

ANALYSIS AND DISCUSSION OF RESULTS

For the case of heat transfer to liquid films about to break down in the case investigated where heating begins at the source of the film to include the initial laminar flow region, no simple mathematical model for analysis can be set up. A formal dimensional analysis can be carried out from the premise:

$$f(Q, \rho, \mu, \sigma, \Delta\sigma, g_x, h_f, h_b) = 0 \qquad (16)$$

to give the basis of an empirical equation of the form

$$\left(\frac{\rho Q}{\mu}\right)_f = K\left(\frac{g_x \mu^4}{\rho \sigma^3}\right)^m \left(\frac{\Delta\sigma}{\sigma_f}\right)^n \left(\frac{h_b}{h_f}\right)^p \qquad (17)$$

However, Portalski [3] has given a non-dimensional group which has definite physical meaning in the wave number (λ/a_0) for isothermal flow. This group can be used as the basis of an empirical equation and its use will facilitate interpretation of results as does the use of simple non-dimensional groups like the Reynolds number and the Froude number.

It is assumed here that the flow pattern as a result of distortion by heat transfer can be represented by the non-dimensional group Φ so that

$$\Phi = \left(\frac{\lambda}{a_0}\right) \exp\left[f\left(\frac{\Delta\sigma}{\sigma_f}\right)\right] \qquad (18)$$

where $(\Delta\sigma/\sigma_f)$ is a surface tension difference function related to the difference between the bulk heating temperature t_b and the film inlet temperature t_f.

Hence

$$\Phi = \frac{\lambda}{a_0}\left(1 + f\left(\frac{\Delta\sigma}{\sigma_b}\right) + \left[f\left(\frac{\Delta\sigma}{\sigma_f}\right)\right]^2 \Big/ 2 + \ldots\right). \qquad (19)$$

If the distortion ∇ is defined as the difference between the flow pattern Φ at breakdown and that of isothermal flow (λ/a_0).

Then

$$\left.\begin{aligned} \nabla &= \Phi - \left(\frac{\lambda}{a_0}\right) \\ \text{and} \\ \nabla &= \frac{\lambda}{a_0}\left\{f\left(\frac{\Delta\sigma}{\sigma_f}\right) + \left[f\left(\frac{\Delta\sigma}{\sigma_f}\right)\right]^2\Big/2 + \ldots\right\}. \end{aligned}\right\} \quad (20)$$

Neglecting second and higher orders of $f(\Delta\sigma/\sigma_f)$ and assuming $f(\Delta\sigma/\sigma_f) = K(\Delta\sigma/\sigma_f)$,

$$\nabla = K\left(\frac{\lambda}{a_0}\right)\left(\frac{\Delta\sigma}{\sigma_f}\right). \quad (21)$$

The distortion ∇ in the region of breakdown is due strictly to the surface tension difference forces arising from temperature differences in the film surface. It follows that the temperature difference $(t_b - t_f)$ implicit in equation (21) is not sufficient to make equation (21) a general expression. Cognizance must be given to the different temperature gradients in the heating fluid and the film that can possibly exist for the same value of $(t_b - t_f)$ for they affect the value of the temperatures in the surface. Introducing the group (h_b/h_f) to adjust the surface tension difference group $(\Delta\sigma/\sigma_f)$ to actual circumstances gives:

$$\nabla = C\left(\frac{\lambda}{a_0}\right)\left(\frac{\Delta\sigma}{\sigma_f}\right)\left(\frac{h_b}{h_f}\right)^p. \quad (22)$$

When $(\lambda/a_0)(\Delta\sigma/\sigma_f)$ is plotted against $4(\rho Q/\mu)_f$ on log-log scales for each of the four different heating temperatures of 140°F, 160°F, 180°F and 200°F, it is suggested from Fig. 10 that

$$\nabla = A\left[\left(\frac{4\,\rho Q}{\mu}\right)_f\right]^n. \quad (23)$$

Combining equations (21) and (23) the following relationship is obtained

$$B\left(\frac{\lambda}{a_0}\right)\left(\frac{\Delta\sigma}{\sigma_f}\right) = \left[4\left(\frac{\rho Q}{\mu}\right)_f\right]^n. \quad (24)$$

Equation (22) shows that the coefficient B depends upon the group (h_b/h_f).

In Fig. 12,

$$D\left(\frac{h_b}{1000}\right)^p\left(\frac{\lambda}{a_0}\right)^p\left(\frac{\Delta\sigma}{\sigma_f}\right)$$

is plotted against $4(\rho Q/\mu)_f$ on log-log scales where the values of D, p and the slope n of the best curve are obtained by the method of least squares, to give the following equation:

$$0{\cdot}0058\left(\frac{h_b}{1000}\right)^{6\cdot89}\left(\frac{\lambda}{a_0}\right)\left(\frac{\Delta\sigma}{\sigma_f}\right) = \left[4\left(\frac{\rho Q}{\mu}\right)_f\right]^{-0\cdot68}. \quad (25)$$

This equation (25) correlates all experimental data to the single curve within $\pm 10\%$. The value of B for each heating condition can now be

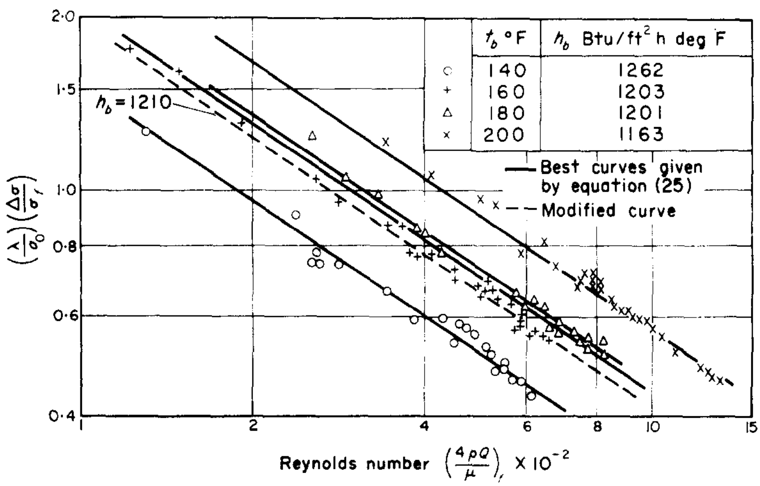

FIG. 10. Variation of $(\lambda/a^{\circ})(\Delta\sigma/\sigma_f)$ with $(4\rho Q/\mu_f)$ for different heat-transfer coefficients of heating liquid.

Table 1

Bulk heating temp. (°F)	140	160	180	200
Experimental heat-transfer coefficient (h_b Btu/ft²h degF)	1262	1203	1201	1163
B	0·0289	0·0208	0·0205	0·0165

obtained for the four single curves in Fig. 10, and is listed in Table 1.

It is apparent from the disposition of the experimental points about each of the single curves in Fig. 10 that B is essentially dependent upon the value of the heat-transfer coefficient h_b on the heating side, and the average value of the film coefficient h_f must be sensibly constant at the breakdown flow rate for all temperature conditions. The coefficient of equation (25), 0·0058, incorporates this constant but unknown coefficient, and the group (h_b/1000) is made non-dimensional by giving the arbitrary denominator the same units as h_b.

It is noted in the case of the bulk heating temperature of 160°F that a value for h_b of 1210 is more satisfactory than the experimental value 1203, a difference of less than 1% (Fig. 10). It is difficult to assess whether this is due to the use of the Dittus–Boelter equation to determine h_b or to experimental error; however, it is apparent that the position of the curve is sensitive to values of h_b.

As the bulk heating temperature t_b has been used as a reference the value of the index 6·89 in equation (25) associated with the film coefficient h_b shows the degree to which actual film surface temperatures and subsequent surface tension difference forces are affected by it. The effect on actual film surface temperatures is an aspect of the established fact that wave motion and rippling give rise to increased heat-transfer rates which Portalski [11] mentions could be largely due to the formation of eddies in liquid films causing bulk mixing. Hsu *et al.* [8] also show that surface temperature differences causing film breakdown are very sensitive to heat flux; their experimental results in Fig. 11 indicate the product of the heat flux and the actual length of the distorted film to the dry patch is sensibly constant for each set of readings of constant Reynolds number.

Further to the effect of heat flux, the results shown in Fig. 12 support the assumption that distortion leading to film breakdown does not alone depend upon the difference between the [illegible] defined by $(\Delta\sigma/\sigma_f)$. The surface tension difference forces in the film surface producing distortion are also related to the wave pattern as described by the wave number (λ/a_0), the cause of uneven heating.

In addition to the fundamental hydrodynamic behaviour of the film there is a secondary effect which influences the flow pattern and subsequently the surface tension difference forces. Distortion causes variations in the local value of Reynolds number which determines the position of the inception of waves as shown in Fig. 6. Irregularity produced in the line of inception produces further irregular behaviour in the flow pattern. This is similar to the effect of eccentricity referred to in the section on experimental work.

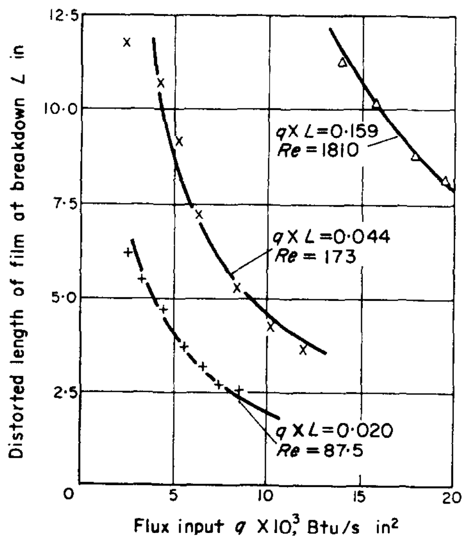

FIG. 11. Effect of heat flux on position of dry patch for different flow rates.

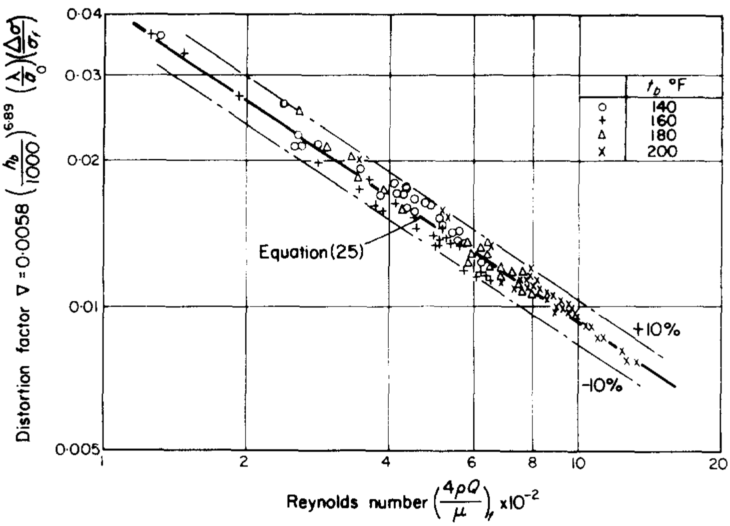

FIG. 12. Variation of distortion factor ∇ at breakdown of falling liquid film due to surface tension difference forces.

The movement of the line of inception of waves with distortion and changes in the mean value of Reynolds number can account for the differences between the minimum wetting rates determined and those of Norman and McIntyre [7]. Breakdown occurs soon after the inception of waves, so before it can occur in a length of 3¼ in the mean value of Reynolds number must be low enough and the distortion such as to cause waves to appear within the 3¼ in of tubing. It is indicated in Fig. 9 that breakdown occurs at higher rates of flow and this is because the dry patch forms outside this length.

It has been assumed throughout that the isothermal flow pattern can be described by the wave number (λ/a_0) which is not entirely true. This assumption partly explains why values of $(\lambda/a_0)(\Delta\sigma/\sigma_f)$ decrease with Reynolds number, in Fig. 10. Portalski gives the solution for the differential equation (6) when it includes second-order terms as

$$a = a_0\left[1 + \alpha \sin n(x - kt) + 0{\cdot}28\,\alpha^2 \cos 2n(x - kt) - \frac{g_x^2}{4a_0\delta n^3}\alpha^2 \sin 2n(x - kt) + \dots\right] \quad (26)$$

where the coefficient of $\sin 2n(x - kt)$ is significant for low values of n, though it decreases rapidly with increasing values of n. The $\sin 2n(x - kt)$ term is present as a result of the lack of balance between the inertia and viscous forces causing coalescing of waves. Then the effective values of $(\lambda/a_0)(\Delta\sigma/\sigma_f)$ at low Reynolds numbers could well be less than those shown in Fig. 10 because of the more disturbed film surface.

The phenomenon of breakdown of liquid films can be explained in thermodynamic terms. Surface energy at a high potential is introduced by water from an upper reservoir and that at a lower potential is rejected into a lower reservoir after heating. Then the function $(\Delta\sigma/\sigma_f)$ defines the maximum available energy for work that can be utilized to distort and breakdown the film. Much of it is realized when a dry patch is about to form on the tube surface as the liquid in the thinned region is close to the bulk heating temperature and the surface temperature in the thickest part of the roped film is comparatively near the film inlet temperature. At onset of distortion due to a given value of $(\Delta\sigma/\sigma_f)$ the wave number (λ/a_0) determines the actual surface tension difference forces which do the work of distorting the film. For breakdown the wavelength λ and the film of mean thickness a_0 must be critically related. If the value of (λ/a_0) is less than the critical value, i.e. when the value of Reynolds number is higher than that given by the minimum wetting rate, the greater frequency of the waves does not produce a sufficient temperature difference to cause enough distortion of the thicker film to break it down.

Thermodynamic reasons suggest that no matter the value of the heat flux input the amount of work required just to break down the film should be a constant for a given Reynolds number of film flow. The experimental results of Hsu *et al.* (Fig. 11) indicate this. They infer that for each flux case where the flow rate is the same the surface tension difference forces in the film surface do the same work when acting on the same fundamental flow pattern because the total energy input, $q \times L$ Btu/s in, is a constant.

If other liquids are considered it can be seen from the definition of (λ/a_0) and the coefficient of $\sin 2n(x - kt)$ in equation (26) that the single curve in Fig. 12 is not going to be applicable. This follows because at the same Reynolds number the flow pattern will be different, giving rise to different surface tension difference forces for the same value of $(\Delta\sigma/\sigma_f)$.

CONCLUSION

The breakdown of liquid films during heat transfer is related to surface tension difference forces set up in the film due to uneven heating caused by a complicated wave flow pattern. For water, a correlation has been obtained from which it is possible to predict minimum wetting rates, but as the wave pattern is not solely a function of Reynolds number, minimum wetting rates for other liquids will be different.

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Résumé—Cet article a pour objet l'écoulement sous l'effet de la pesanteur d'un film liquide sous-refroidi et sa rupture pendant le transport de chaleur en tant qu'étude préliminaire à celle du "coup de feu" dans le cas de l'écoulement diphasique. Les modifications hydrodynamiques du film dues à des différences de tension superficielle sont considérées à l'aide d'une analyse mathématique rigoureuse pour des systèmes isothermes. Une corrélation, faisant intervenir une expression mathématique du nombre d'onde (λ/a_0) pour l'écoulement isotherme, est obtenue dans le but de prédire les débits d'eau lorsque la rupture se produit pour des conditions données de chauffage:

$$0{,}0058 \left(\frac{\lambda}{a_0}\right) \left(\frac{\Delta\sigma}{\sigma_f}\right) \left(\frac{h_b}{1000}\right)^{6{,}8} = \left(\frac{4\rho Q}{\mu}\right)_f^{-0{,}68}$$

Zusammenfassung—Als vorläufige Studie zur Untersuchung des burnout in der Zweiphasenströmung wird ein unterkühlter fallender Flüssigkeitsfilm und sein abreissen während des Wärmeübergangs untersucht. Die hydrodynamischen Veränderungen des Films infolge unterschiedlicher Oberflächenspannungen werden mit Hilfe einer streng mathematischen Analysis für das isotherme System berücksichtigt. Eine Beziehung die den mathematischen Ausdruck für die Wellenzahl (λ/a_0) der isothermen

Strömung einschliess wurde aufgestellt um den Durchsatz von Wasser beim Abreissen für gegebene gegebene Heizbedingungen zu ermitteln; sie lautet

$$0{,}0058 \left(\frac{\lambda}{a_0}\right) \left(\frac{\Delta\sigma}{\sigma_f}\right) \left(\frac{h_b}{1000}\right)^{6{,}8} = \left(\frac{4\rho Q}{\mu}\right)_f^{-0{,}68}$$

Аннотация—В статье разбирается случай падающей пленки недогретой жидкости и нарушение режима её течения под воздействием тепловых потоков. Это исследование поставлено в связи с явлением выгорания в случае двухфазного течения. На основе тщательного математического анализа изотермических систем рассмотрены гидродинамические изменения пленки, обусловленные разностью сил поверхностного натяжения. Для данных условий нагрева установлено соотношение, связывающее расход воды с волновым числом (λ/a_0) изотермического течения, в виде

$$0{,}0058 \left(\frac{\lambda}{a_0}\right) \left(\frac{\Delta\sigma}{\sigma_f}\right) \left(\frac{h_b}{1000}\right)^{6{,}89} = \left(\frac{4\rho Q}{\mu}\right)_f^{-0{,}68}$$